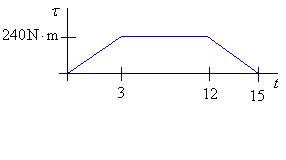
**Example**

Suppose you rotate a merry-go-round CCW (radius R = 1.2m) with a force of 200N applied tangentially to the edge for 15s. What angular impulse do you give the merry-go-round? Suppose your force steadily increases from 0 to 200N in 3s, remains constant there for 10s, and then decreases to 0 again in 2s. What is the angular impulse delivered in this case?

From the equation, it would be:



In the second case, the torque applied would go from 0 to τ = rFsinφ = (1.2)(200)sin90˚ = 240N·m by t = 3s. Remain there until t = 10s, and then go back to 0. Plotted out, it would look like,



The net angular impulse would just be the area under the τ-t curve. So



**Example: Angular momentum of an electron**

Consider an electron (me = 9.11×10-31kg) counter-clockwise orbiting a proton. If its oribital radius is 0.1nm, and its speed is 2×106m/s, what is its angular momentum about the nucleus?

Well, **L** = I**ω**. The moment of inertia for a point mass rotating in a circle of radius r is I = mr2. And its angular velocity is ω = v/r. So we have,



This is the general expression for the angular momentum of a particle rotating in a circle with radius r and speed v. So this comes to:



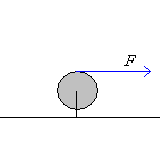
4. Suppose you are rotating a merry-go-round (m = 100kg, r = 1.5m) with a force that varies with time (since you’re getting tired) according to: . What is the angular velocity of the merry-go-round after 10s? You can treat the merry-go-round as a hoop.

Using the impulse-momentum equation…



**Example: Rotating disk**

Suppose that a grindstone, which we’ll approximate as a solid disk, is initially at rest. Then suppose we exert a constant force of 25N tangent to the edge of the grindstone (as illustrated below). If we exert the force for 5s, what angular velocity will the grindstone acquire? Assume the radius is 30cm, and the mass is 10kg. Suppose that the torque varies with time according to the plot above. Then what will be the angular velocity?

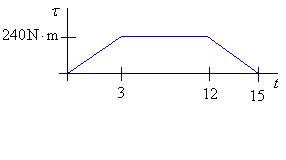


Applying the rotational impulse-momentum equation we have,



and so the angular velocity will be 83.3 rad/s in the CW direction, as is obvious from the picture.

If the torque varies according to:



The net angular impulse would just be the area under the τ-t curve. So



and using the angular impulse-momentum equation, we can determine the final angular velocity of the grind stone according to:



9. A playground carousel with moment of inertia 150 kg·m2 is rotating counterclockwise about its center on frictionless bearings. The carousel has a radius of 1.5m, and an initial angular speed of 1.4 rev/s. A 40kg person standing still on the ground grabs onto one of the bars on the carousel very close to its outer edge and climbs aboard. Find the final angular speed (in rev/s) of the carousel after the person climbs aboard.

Applying rotation IM equation, and realizing that no external torque means L is conserved we get:



which works out to:



**Example: Merry-go-round**

Suppose you (m = 60kg) are on the rim of a merry-go-round (shaped like a hoop with M = 100kg, R = 1.5m) making one revolution every second. If you walk to the center of the merry-go-round, what will be the new rate of rotation? What will be the change in KE? Assume that the axle upon which the merry-go-round turns is frictionless so that it will not exert a torque.

Since there will be no net torque acting on our system of objects then, we’ll have,



The initial angular momentum is that of the merry-go-round and the person standing on the edge. The final is that of the merry-go-round alone, since the person will have moved to the center and no longer be rotating. So we have,



To find the frequency of rotation, we divide by 2π, which is the number of radians per revolution.



So the rate of revolution increases to 1.6 revolutions per second. Now the change in KE is:



So there is an increase in KE. Where does this extra energy come from? It comes from the work done by the merry-go-round on you moving you towards the center.

**Example: Collapsing Star**

When stars run out of hydrogen to burn, their gravitational pull often collapses them into a much smaller radius. Suppose our Sun, which is now rotating about itself once every approximately 25days (according to Wikipedia), was to collapse to 1/100 its present radius to form a dwarf star. What would be its new rotation rate?

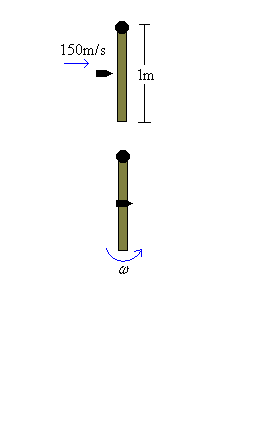
We will use conservation of angular momentum again of course. Note that I for a sphere rotating about its center is 2/5MR2.



So the angular velocity will increase by a factor of 10 000. Therefore the period of rotation will decrease by a factor of 10 000. So the Sun would rotate once every 25/10 000 = 0.0025 days, which is once every 3.6 minutes.

**Example**

Suppose you fire a bullet (mass 100g) with a velocity of 150m/s into the middle of a door, which weighs 30kg, is 1m wide and 2.5m tall. With what angular velocity will the door open assuming the bullet embeds in the door?

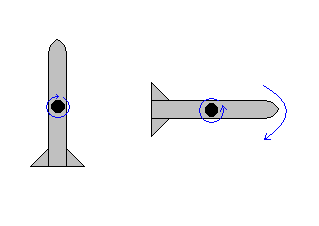


Again use conservation of angular momentum. The angular momentum of the bullet is mvr. Also remember that the moment of inertia of a board rotating about its end point is (1/3)Mℓ2. So we have,



**Example: Gyroscope**

Suppose you have a rocket ship (M = 10 000kg, h = 25m) that needs to be turned to the right. You have a wheel (m = 100kg) with radius R = 20cm, spinning about the axis pointing into the page at a rate of ω = 5 rev/s = 10π rad/s. Since you want to turn the rocket to the right, you flip the wheel upside down so that it now rotates about the axis pointing up from the ground. What will be the rate of rotation of the rocket about itscenter of mass? When will it complete a 90˚ rotation? Approximate the rocket as board with mass M and length h.

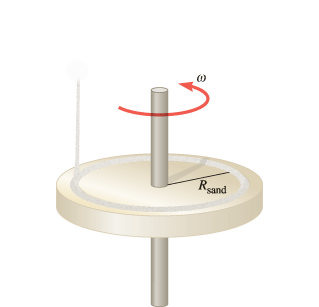


Since the initial angular momentum is going into the page, the final angular momentum must be as well – due to conservation of momentum. Thus, if we flip the wheel so that has the oppositte angular momentum, the rocket must start to rotate to keep the net angular momentum the same. To determine the rate, we’ll use conservation of angular momentum,



So the rocket will turn clockwise at a rate of 9.65×10-4 rad/s. Ths correlates to about 3.3˚ per minute. So it would take about 30 minutes to complete a 90˚ turn.

6. A solid disk rotates in the horizontal plane at an angular velocity of 2 rad/s with respect to an axis perpendicular to the disk at its center. The moment of inertia of the disk is 1.2 kg·m2. From above, sand is dropped straight down onto this rotating disk, so that a thin uniform ring of sand is formed at a distance 0.6 m from the axis. The sand in the ring has a mass of 0.45 kg. After all the sand is in place, what is the angular velocity of the disk?



Formally, we would use the impulse-momentum equation on the disk+sand. There is no net external angular impulse acting on them to make them rotate about the axis, therefore angular momentum is conserved. So we have:



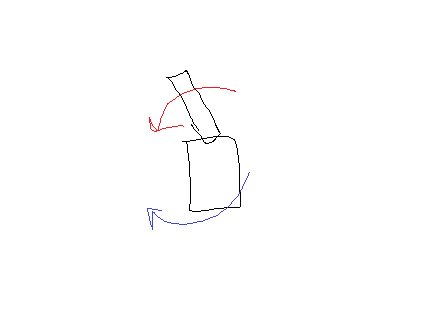
2. Suppose you’re sitting on a chair with your arms outstretched each holding 10kg weights. And suppose you set yourself spinning at a rate of ω = 1.2 rad/s. If you pull your arms halfway in, what will be your new angular velocity? Only consider the moment of inertia due to the weights.

Use conservation of angular momentum,

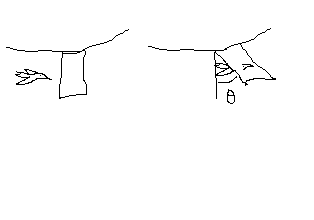


**Example**

Suppose have velociraptor trying to make a sudden turn. How fast must swing tail to make rotation of body by 90 degrees or something?



**Question 6.** A 75g bird flying at 20m/s becomes embedded in the middle of a very starchy towel (i.e. stiff like a board). Say the towel has a mass m = 1.2kg, and length 80cm. What maximum angle does the shirt + bird rotate through?



Applying conservation of angular momentum to the collision we have:

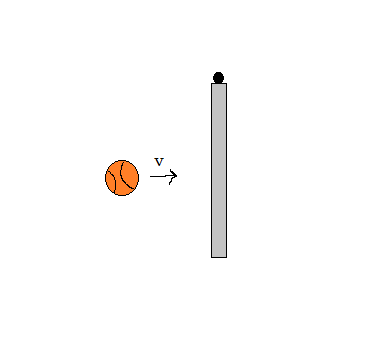


Then to determine the max angle we use conservation of energy:



**Example**

Suppose you throw a basketball at a door. Let basketball have mass mball = 1.2kg, and door have mass mdoor = 40kg, and length 1.3m. Suppose basketball is thrown at midpoint of door with velocity v = 16m/s, and rebounds elastically. With what angular velocity will the door open?



Have conservation of angular momentum:



And then conservation of energy implies:



Want to solve these equations for ω2. So solve top for ω1 = (12.5 – 22.5ω2)/0.507. Plugging this into the next equation we have:

307 = 0.507[(12.5-22.5ω2)/0.507]2 + 22.5ω22

307 = (12.5-22.5ω2)2/0.507 + 22.5ω22.

Solution is ω2 = 1.09 rad/s, and ω1 = -12 rad/s, which correlates to v = (12)(1.3/2) = 7.8m/s, backwards of course.

**Question 8**. A 2.5m diameter merry-go-round with a mass of 225kg is spinning at 15 rpm. Fred (John’s brother) runs onto the merry-go-round at 7m/s, in the same direction it is rotating. Fred’s mass is m = 67kg. What is the new rate of rotation of the merry-go-round in rpm now? You may treat the merry-go-round as a disk.

We have the angular impulse-momentum equation Li + K = Lf. The angular impulse K = 0. And so angular momentum is conserved. And so we have:

